F08FGFP (PDORMTR)

NAG Parallel Library Routine Document

Note: before using this routine, please read the Users' Note for your implementation to check for implementation-dependent details. You are advised to enclose any calls to NAG Parallel Library routines between calls to Z01AAFP and Z01ABFP.

1 Description

F08FGFP (PDORMTR) multiplies an m by n real matrix C_s by a real orthogonal matrix Q, where C_s is a submatrix of a larger m_C by n_C matrix C, i.e.,

$$C_s(1:m,1:n) \equiv C(i_C:i_C+m-1,j_C:j_C+n-1).$$

Note: if $i_C = j_C = 1$, $m = m_C$ and $n = n_C$, then $C_s \equiv C$.

The matrix Q is defined by the factorization, $A_s = QTQ^T$ with A_s real symmetric, Q orthogonal and T real symmetric tridiagonal, as computed by the routine F08FEFP (PDSYTRD).

This routine may be used to form one of the products

$$QC_s, Q^TC_s, C_sQ$$
 or C_sQ^T ,

without explicitly forming Q. The result is overwritten on C_s .

F08FGFP (PDORMTR) can be used to compute the eigenvectors of A following the computation of the eigenvectors of T by F08JKFP (PDSTEIN).

2 Specification

```
SUBROUTINE FO8FGFP(SIDE, UPLO, TRANS, M, N, A, IA, JA, IDESCA,

TAU, C, IC, JC, IDESCC, WORK, LWORK, INFO)

ENTRY PDORMTR(SIDE, UPLO, TRANS, M, N, A, IA, JA, IDESCA,

TAU, C, IC, JC, IDESCC, WORK, LWORK, INFO)

DOUBLE PRECISION A(*), TAU(*), C(*), WORK(*)

INTEGER M, N, IA, JA, IDESCA(*), IC, JC, IDESCC(*),

LWORK, INFO

CHARACTER*1 SIDE, UPLO, TRANS
```

The ENTRY statement enables the routine to be called by its ScaLAPACK name.

3 Usage

3.1 Definitions

The following definitions are used in describing the data distribution within this document:

numroc $(\hat{\ell}, L_b^X, p, s^X, \ell_p)$ — a function which gives the **num**ber of **rows or columns** of a distributed matrix X owned by the processor with the row or column coordinate p $(p_r \text{ or } p_c)$, where $\hat{\ell}$ is the total number of rows or columns of the matrix, L_b^X is the blocking factor used $(M_b^X \text{ or } N_b^X)$, s^X is the row or column coordinate $(s_r^X \text{ or } s_c^X)$ of the processor that possesses the first row or column of the distributed matrix and ℓ_p is either m_p or n_p . The Library provides the function Z01CAFP (NUMROC) for the evaluation of this

function.

 $\mathrm{indxg2p}(k,L_b^X,p,s^X,\ell_p)$ — a function which gives the processor row or column coordinate which

possess the row or column index k of a distributed matrix. The arguments L_b^X , s^X and ℓ_p have the same meaning as in the function numroc. However, the argument p is a dummy integer. The Library provides the utility function Z01CDFP (INDXG2P) for the evaluation

of this function.

iclm(i,j) – a function which gives the least common multiple (LCM) of the two

integers i and j.

3.2 Global and Local Arguments

The following global **input** arguments must have the same value on entry to the routine on each processor and the global **output** arguments will have the same value on exit from the routine on each processor:

Global input arguments: SIDE, UPLO, TRANS, M, N, IA, JA, IC, JC, the array elements

IDESCA(1), IDESCA(3:8), IDESCC(1) and IDESCC(3:8)

Global output arguments: INFO

The remaining arguments are local.

3.3 Distribution Strategy

On entry, the matrix C must be stored in the cyclic two-dimensional block format and its descriptor array IDESCC must contain the details of the distributed matrix. The indices i_C and j_C and the dimensions m and n identify the submatrix C_s . See the F08 Chapter Introduction for further details.

It is assumed that the data has already been correctly distributed, and if this is not the case, then this routine will fail to produce correct results.

3.4 Related Routines

The Library provides many support routines for the generation/distribution and input/output of data. The following routines may be used in conjunction with F08FGFP (PDORMTR):

Real matrix output: X04BFFP
Real matrix generation: F01ZQFP
Real vector scatter: F01ZPFP
Real matrix gather: F01WAFP

4 Arguments

1: SIDE — CHARACTER*1

Global Input

On entry: indicates how Q or Q^T is to be applied to C_s as follows:

if SIDE = 'L', Q or Q^T is applied to C_s from the left; if SIDE = 'R', Q or Q^T is applied to C_s from the right.

Constraint: SIDE = 'L' or 'R'.

2: UPLO — CHARACTER*1

Global Input

On entry: indicates whether the upper or lower triangular part of A_s is stored, as follows:

if UPLO = 'U', the upper triangular part of A_s is stored; if UPLO = 'L', the lower triangular part of A_s is stored.

Constraint: UPLO = 'U' or 'L'.

3: TRANS — CHARACTER*1

Global Input

On entry: indicates whether Q or Q^T is to be applied to C_s as follows:

if TRANS = 'N', Q is applied to C_s ; if TRANS = 'T', Q^T is applied to C_s .

Constraint: TRANS = 'N' or 'T'.

4: M — INTEGER

Global Input

On entry: m, the number of rows of the matrix C_s .

Constraint: $M \ge 0$

5: N — INTEGER

Global Input

On entry: n, the number of columns of the matrix C_s .

Constraint: $N \ge 0$

6: A(*) — DOUBLE PRECISION array

Local Input

Note: array A is formally defined as a vector. However, you may find it more convenient to consider A as a two-dimensional array of dimension (IDESCA(9), γ), where

```
if SIDE = 'L', \gamma \ge \text{numroc}(\text{JA+M-1,IDESCA}(6), p_c, \text{IDESCA}(8), n_p)
if SIDE = 'R', \gamma \ge \text{numroc}(\text{JA+N-1,IDESCA}(6), p_c, \text{IDESCA}(8), n_p).
```

On entry: details of the vectors which define the elementary reflectors as returned by a call to F08FEFP (PDSYTRD).

7: IA — INTEGER

Global Input

On entry: i_A , the row index of the matrix A that identifies the first row of the submatrix A_s to be factorized.

8: JA — INTEGER

Global Input

On entry: j_A , the column index of the matrix A that identifies the first column of the submatrix A_s to be factorized.

Constraint: mod(IA-1, IDESCA(5)) = mod(JA-1, IDESCA(6)) = 0.

9: IDESCA(*) — INTEGER array

Local Input

Note: the dimension of the array IDESCA must be at least 9.

Distribution: the array elements IDESCA(1) and IDESCA(3),...,IDESCA(8) must be global to the Library Grid and the elements IDESCA(2) and IDESCA(9) are local to each processor.

On entry: the description array for the matrix A. This array must contain details of the distribution of the matrix A and the logical processor grid.

IDESCA(1), the descriptor type. For this routine, which uses a cyclic two-dimensional block distribution, IDESCA(1) = 1;

IDESCA(2), the Library context, usually returned by a call to the Library Grid initialisation routine Z01AAFP;

IDESCA(3), the number of rows, m_A , of the matrix A;

IDESCA(4), the number of columns, n_A , of the matrix A;

IDESCA(5), the blocking factor, M_b^A , used to distribute the rows of the matrix A;

IDESCA(6), the blocking factor, N_b^A , used to distribute the columns of the matrix A;

IDESCA(7), the processor row index, s_r^A , over which the first row of the matrix A is distributed;

IDESCA(8), the processor column index, s_c^A , over which the first column of the matrix A is distributed;

IDESCA(9), the leading dimension of the conceptual two-dimensional array A.

Constraints:

```
IDESCA(1) = 1;

IDESCA(3) \geq 0; IDESCA(4) \geq 0;

IDESCA(5) = IDESCA(6) \geq 1;

0 \leq \text{IDESCA}(7)} \leq m_p - 1; 0 \leq \text{IDESCA}(8) \leq n_p - 1;

if SIDE = 'L', IDESCA(9) \geq \max(1, \text{numroc}(\text{IA}+\text{M}-1, \text{IDESCA}(5), p_r, \text{IDESCA}(7), m_p));

if SIDE = 'R', IDESCA(9) \geq \max(1, \text{numroc}(\text{IA}+\text{N}-1, \text{IDESCA}(5), p_r, \text{IDESCA}(7), m_p)).
```

10: TAU(*) — DOUBLE PRECISION array

Local Input

Note: the dimension of the array TAU must be at least α where

```
if SIDE = 'L' and UPLO = 'U', \alpha = \text{numroc}(\text{IDESCA}(3), \text{IDESCA}(6), p_c, \text{IDESCA}(8), n_p); if SIDE = 'L' and UPLO = 'L', \alpha = \text{numroc}(\text{JA+M-2}, \text{IDESCA}(6), p_c, \text{IDESCA}(8), n_p); if SIDE = 'R' and UPLO = 'U', \alpha = \text{numroc}(\text{IDESCA}(4), \text{IDESCA}(6), p_c, \text{IDESCA}(8), n_p); if SIDE = 'R' and UPLO = 'L' \alpha = \text{numroc}(\text{JA+N-2}, \text{IDESCA}(6), p_c, \text{IDESCA}(8), n_p)
```

On entry: details of the elementary reflectors, as returned by a call to F08FEFP (PDSYTRD).

11: C(*) — DOUBLE PRECISION array

Local Input/Local Output

Note: array C is formally defined as a vector. However, you may find it more convenient to consider C as a two-dimensional array of dimension (IDESCC(9), γ), where $\gamma \geq \text{numroc}(\text{JC+N-1},\text{IDESCC}(6),p_c,\text{IDESCC}(8),n_p)$.

On entry: the local part of the matrix C.

On exit: overwritten by QC_s , Q^TC_s , C_sQ or C_sQ^T .

12: IC — INTEGER

Global Input

On entry: i_C , the row index of matrix C that identifies the first column of the submatrix C_s .

Constraint: $1 \le IC \le IDESCC(3) - M + 1$.

13: JC — INTEGER

Global Input

On entry: j_C , the column index of matrix C that identifies the first column of the submatrix C_s .

Constraint: $1 \le JC \le IDESCC(4) - N + 1$.

14: IDESCC(*) — INTEGER array

 $Local\ Input$

Note: the dimension of the array IDESCC must be at least 9.

Distribution: the array elements IDESCC(1) and IDESCC(3),...,IDESCC(8) must be global to the processor grid and the elements IDESCC(2) and IDESCC(9) are local to each processor.

On entry: the description array for the matrix C. This array must contain details of the distribution of the matrix C and the logical processor grid.

IDESCC(1), the descriptor type. For this routine, which uses a cyclic two-dimensional block distribution, IDESCC(1) = 1;

IDESCC(2), the Library context, usually returned by a call to the Library Grid initialisation routine Z01AAFP;

IDESCC(3), the number of rows, m_C , of the matrix C;

IDESCC(4), the number of columns, n_C , of the matrix C;

IDESCC(5), the blocking factor, M_b^C , used to distribute the rows of the matrix C;

IDESCC(6), the blocking factor, N_b^C , used to distribute the columns of the matrix C;

IDESCC(7), the processor row index, s_r^C , over which the first row of the matrix C is distributed;

IDESCC(8), the processor column index, s_c^C , over which the first column of the matrix C is distributed;

IDESCC(9), the leading dimension of the conceptual two-dimensional array C.

Constraints:

```
\begin{split} &\text{IDESCC}(1) = 1; \\ &\text{IDESCC}(2) = \text{IDESCA}(2); \\ &\text{IDESCC}(3) \geq 0; \\ &\text{IDESCC}(4) \geq 0; \\ &\text{IDESCC}(5) \geq 1; \\ &\text{IDESCC}(6) \geq 1; \\ &0 \leq \text{IDESCC}(7) \leq m_p - 1; \\ &0 \leq \text{IDESCC}(7) \leq m_p - 1; \\ &\text{IDESCC}(9) \geq \max(1, \text{numroc}(\text{IDESCC}(3), \text{IDESCC}(5), p_r, \text{IDESCC}(7), m_p)). \\ &\text{if UPLO} = \text{`U'}, i_{aa} = \text{IA}; j_{aa} = \text{JA} + 1; i_{cc} = \text{IC}; j_{cc} = \text{JC}; \\ \end{split}
```

The following constraints are to ensure common alignments in distribution and blocking between the matrices A and C:

```
if SIDE = 'L', IDESCC(5) = IDESCA(5); \alpha_{c1} = \alpha_{a1}; \beta_{c1} = \beta_{a1}; if SIDE = 'R', IDESCC(6) = IDESCA(5); \alpha_{c2} = \alpha_{a1}.
```

where

```
\alpha_{a1} = \text{mod}(i_{aa}-1, \text{IDESCA}(5));
\alpha_{c1} = \text{mod}(i_{cc}-1, \text{IDESCC}(5));
\alpha_{c2} = \text{mod}(j_{cc}-1, \text{IDESCC}(6));
\beta_{a1} = \text{indxg2p}(i_{aa}, \text{IDESCA}(5), p_r, \text{IDESCA}(7), m_p);
\beta_{c1} = \text{indxg2p}(i_{cc}, \text{IDESCC}(5), p_r, \text{IDESCC}(7), m_p);
```

where

```
if UPLO = 'U', i_{aa} = IA; j_{aa} = JA +1; i_{cc} = IC; j_{cc} = JC; if UPLO = 'L', i_{aa} = IA +1; j_{aa} = JA; if SIDE = 'L', i_{cc} = IC +1; j_{cc} = JC; if SIDE = 'R', i_{cc} = IC; j_{cc} = JC+1;
```

15: WORK(*) — DOUBLE PRECISION array

Local Workspace/Local Output

Note: the dimension of WORK must be at least $\max(1, \text{LWORK})$. The minimum value of LWORK required to successfully call this routine can be obtained by setting LWORK = -1. The required size is returned in array element WORK(1).

On exit: WORK(1) contains the minimum dimension of the array WORK required to successfully complete the task.

16: LWORK — INTEGER

Local Input

On entry: either -1 (see WORK) or the dimension of the array WORK required to successfully complete the task. If LWORK is set to -1 on entry this routine simply performs some initial error checking and then, if these checks are successful, calculates the minimum size of LWORK required.

Constraints:

```
if SIDE = 'L', m_1 = \text{M}-1; \, n_1 = \text{N}; \text{LWORK} = \max(N_b^A*(N_b^A-1)/2, (\mu_{c1}+\mu_{c2})*N_b^A) + N_b^A*N_b^A \text{ or LWORK} = -1;
```

```
if SIDE = 'R', m_1 = M; n_1 = N-1; LWORK = \max(N_b^A * (N_b^A - 1)/2, \gamma_3 * N_b^A) + N_b^A * N_b^A \text{ or LWORK} = -1; where i_{aa}, i_{cc}, \alpha_{a1}, \alpha_{c1}, \alpha_{c2} \text{ and } \beta_{c1} \text{ are defined in the description of the argument IDESCC(9)}; N_b^A = \text{IDESCA}(6); \alpha_{a2} = \text{mod}(j_{aa} - 1, \text{IDESCA}(6)); \mu_{a1} = \text{numroc}(n_1 + \beta_{a1}, \text{IDESCA}(5), p_r, \beta_{a1}, m_p); \beta_{c2} = \text{indxg2p}(j_{cc}, \text{IDESCC}(8), n_p); \mu_{c1} = \text{numroc}(m_1 + \alpha_{c1}, \text{IDESCC}(5), p_r, \beta_{c1}, m_p); \mu_{c2} = \text{numroc}(n_1 + \alpha_{c2}, \text{IDESCC}(6), p_c, \beta_{c2}, n_p); \delta = \text{iclm}(m_p, n_p)/n_p; \gamma_1 = \text{numroc}(n_1 + \alpha_{c2}, \text{IDESCC}(6), 0, 0, n_p); \gamma_2 = \text{numroc}(\gamma_1, \text{IDESCC}(6), 0, 0, \delta); \gamma_3 = \mu_{c2} + \max(\mu_{a1} + \gamma_2, \mu_{c1}).
```

17: INFO — INTEGER

Global Output

The NAG Parallel Library provides a mechanism, via the routine Z02EAFP, to reduce the amount of parameter validation performed by this routine. For a full description refer to the Z02 Chapter Introduction.

On exit: INFO = 0 (or -9999 if reduced error checking is enabled) unless the routine detects an error (see Section 5).

5 Errors and Warnings

If INFO < 0 an explanatory message is output and control returned to the calling program.

```
INFO = -i
```

On entry, one of the arguments was invalid:

```
if the kth argument is a scalar INFO = -k;
if the kth argument is an array and its jth element is invalid, INFO = -(100 \times k + j).
```

This error occurred either because a global argument did not have the same value on all logical processors, or because its value on one or more processors was incorrect.

6 Further Comments

6.1 Algorithmic Detail

See Anderson et al. [1], Golub and Van Loan [2] and Blackford et al. [3].

6.2 Parallelism Detail

The Level-3 BLAS operations are carried out in parallel within the routine.

7 References

- [1] Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia
- [2] Golub G H and van Loan C F (1996) Matrix Computations Johns Hopkins University Press (3rd Edition), Baltimore

[3] Blackford L S, Choi J, Cleary A, D'Azevedo E, Demmel J, Dhillon I, Dongarra J, Hammarling S, Henry G, Petitet A, Stanley K, Walker D and Whaley R C (1997) ScaLAPACK Users' Guide SIAM 3600 University City Science Center, Philadelpia, PA 19104-2688, USA. URL: http://www.netlib.org/scalapack/slug/scalapack_slug.html

8 Example

The example program illustrates the the computation of eigenvectors of a 10 by 10 real symmetric matrix A_s . The (i,j)th element of this matrix A_s is max(i,j). This matrix is generated using routine F01ZQFP.

The real matrix A is reduced to the real tridiagonal form T using F08FEFP (PDSYTRD). The argument UPLO is set to 'U' and hence only the upper triangular part of the matrix A_s is used.

The diagonal vector d and the off-diagonal vector e are gathered to every logical processor using the routine F01ZPFP. The vector d (denoted locally by the array DL) and the the vector e (denoted locally by the array EL) are printed on the root processor.

The eigenvalues of the real tridiagonal matrix T are computed using the routine F08JJFP (PDSTEBZ). Routine F08JKFP (PDSTEIN) gives the eigenvectors of the real tridiagonal matrix.

Finally, the eigenvectors of the original matrix A_s are obtained by calling routine F08FGFP (PDORMTR). The first four computed eigenvectors are gathered to the root processor and printed.

8.1 Example Text

```
FO8FGFP Example Program Text
NAG Parallel Library Release 3. NAG Copyright 1999.
.. Parameters ..
TNTEGER.
                NOUT
PARAMETER
               (NOUT=6)
INTEGER
               N
PARAMETER
               (N=10)
               NA
INTEGER
PARAMETER
                (NA=20)
INTEGER
                MP, NP
               (MP=2,NP=2)
PARAMETER
INTEGER.
                NB
PARAMETER
                (NB=2)
INTEGER
                LDA, TDA
PARAMETER
                (LDA=NA/MP+NB,TDA=NA/NP+NB)
INTEGER
                LWORK, LIWORK
PARAMETER
                (LWORK=100,LIWORK=100)
.. Local Scalars ..
DOUBLE PRECISION ORFAC
INTEGER
               I, IA, ICNTXT, IFAIL, INFO, IZ, J, JA, JZ, M,
                NSPLIT, MPROC, NPROC
LOGICAL
                ROOT
CHARACTER
                SIDE, TRANS, UPLO
.. Local Arrays ..
DOUBLE PRECISION A(LDA, TDA), D(TDA), DL(N), E(TDA), EL(N),
                GAP(MP*NP), TAU(TDA), W(NA), WORK(LWORK), X(N,4),
                Z(LDA,TDA)
INTEGER
                IBLOCK(N), ICLUSTR(MP*NP*2), IDESCA(9),
                IDESCZ(9), ISPLIT(N), IWORK(LIWORK), JFAIL(NA)
.. External Functions ..
LOGICAL
                Z01ACFP
EXTERNAL
                Z01ACFP
.. External Subroutines ..
EXTERNAL.
                F01WAFP, F01ZPFP, F01ZQFP, F08FEFP, F08FGFP,
                FO8JJFP, FO8JKFP, GMATA, ZO1AAFP, ZO1ABFP
```

```
.. Executable Statements ..
ROOT = ZO1ACFP()
IF (ROOT) THEN
   WRITE (NOUT,*) 'FO8FGFP Example Program Results'
   WRITE (NOUT,*)
END IF
MPROC = MP
NPROC = NP
IFAIL = 0
CALL ZO1AAFP(ICNTXT, MPROC, NPROC, IFAIL)
Set the starting address and array descriptor for A
IA = 1
JA = 1
IDESCA(1) = 1
IDESCA(2) = ICNTXT
IDESCA(3) = NA
IDESCA(4) = NA
IDESCA(5) = NB
IDESCA(6) = NB
IDESCA(7) = 0
IDESCA(8) = 0
IDESCA(9) = LDA
Generate the matrix A
IFAIL = 0
CALL FO1ZQFP(GMATA, N, N, A, IA, JA, IDESCA, IFAIL)
Reduce A to tridiagonal form
UPLO = 'U'
CALL FO8FEFP (UPLO, N, A, IA, JA, IDESCA, D, E, TAU, WORK, LWORK, INFO)
Gather the diagonal D to each logical processor
IFAIL = 0
CALL FO1ZPFP(N,IA,JA,IDESCA,D,DL,WORK,LWORK,IFAIL)
Gather the off-diagonal E to each logical processor
IFAIL = 0
CALL FO1ZPFP(N,IA,JA,IDESCA,E,EL,WORK,LWORK,IFAIL)
Compute the eigenvalues of T (and of A)
IF (UPLO.EQ.'L') THEN
   CALL FO8JJFP(ICNTXT,'A','B',N,0.0D0,0.0D0,0,0,-1.0D0,DL,EL,M,
                NSPLIT, W, IBLOCK, ISPLIT, WORK, LWORK, IWORK, LIWORK,
                INFO)
ELSE IF (UPLO.EQ.'U') THEN
   CALL F08JJFP(ICNTXT, 'A', 'B', N, 0.0D0, 0.0D0, 0, 0, -1.0D0, DL, EL(2),
                M, NSPLIT, W, IBLOCK, ISPLIT, WORK, LWORK, IWORK, LIWORK,
                INFO)
END IF
```

```
Print the computed eigenvalues
   IF (ROOT) THEN
      WRITE (NOUT,*) 'Eigenvalues'
      WRITE (NOUT,*)
      DO 20 I = 1, N
         WRITE (NOUT, '(1X, I3, 5X, F12.4)') I, W(I)
20
      CONTINUE
  END IF
  Set the starting address and array descriptor for the matrix of
  eigenvectors, Z
   IZ = 1
   JZ = 1
   IDESCZ(1) = 1
  IDESCZ(2) = ICNTXT
  IDESCZ(3) = NA
  IDESCZ(4) = NA
  IDESCZ(5) = NB
   IDESCZ(6) = NB
   IDESCZ(7) = 0
   IDESCZ(8) = 0
  IDESCZ(9) = LDA
  Compute the eigenvectors of T
  ORFAC = 1.0D-06
  IF (UPLO.EQ.'L') THEN
      CALL FO8JKFP(N,DL,EL,M,W,IBLOCK,ISPLIT,ORFAC,Z,IZ,JZ,IDESCZ,
                   WORK, LWORK, IWORK, LIWORK, JFAIL, ICLUSTR, GAP, INFO)
  ELSE IF (UPLO.EQ.'U') THEN
    CALL FO8JKFP(N,DL,EL(2),M,W,IBLOCK,ISPLIT,ORFAC,Z,IZ,JZ,IDESCZ,
                   WORK, LWORK, IWORK, LIWORK, JFAIL, ICLUSTR, GAP, INFO)
  END IF
  Transform the eigenvectors of T to those of A
  SIDE = 'L'
  TRANS = 'N'
  CALL FO8FGFP(SIDE, UPLO, TRANS, N, N, A, IA, JA, IDESCA, TAU, Z, IZ, JZ,
                IDESCZ,WORK,LWORK,INFO)
  Gather the first four eigenvectors to the (root) processor (0,0)
   as the matrix X
  IFAIL = 0
  CALL FO1WAFP(N,4,Z,IZ,JZ,IDESCZ,0,0,X,N,WORK,LWORK,IFAIL)
  Print these four eigenvectors
   IF (ROOT) THEN
      WRITE (NOUT,*) 'The first four eigenvectors'
      WRITE (NOUT,*)
      DO 40 I = 1, N
         WRITE (NOUT, (1X, 4(D15.5, 1X)))) (X(I, J), J=1, 4)
40
      CONTINUE
  END IF
```

```
*
     IFAIL = 0
     CALL ZO1ABFP(ICNTXT, 'N', IFAIL)
     STOP
     END
     SUBROUTINE GMATA(I1,I2,J1,J2,AL,LDAL)
     GMATA generates the block A(I1: I2, J1: J2) of the upper
     triangular part of the symmetric matrix A such
     that
        a(i,j) = max(i,j)
     in the array AL.
     .. Scalar Arguments ..
     INTEGER
                     I1, I2, J1, J2, LDAL
     .. Array Arguments ..
     DOUBLE PRECISION AL(LDAL,*)
     .. Local Scalars ..
                     I, J, K, L
     INTEGER
     .. Intrinsic Functions ..
     INTRINSIC MAX
     .. Executable Statements ..
     L = 1
     DO 40 J = J1, J2
        K = 1
        D0 20 I = I1, I2
           IF (I.LE.J) THEN
              AL(K,L) = MAX(I,J)
           END IF
           K = K + 1
       CONTINUE
  20
        L = L + 1
  40 CONTINUE
     RETURN
     END
```

8.2 Example Data

None.

8.3 Example Results

FO8FGFP Example Program Results

Eigenvalues

| 1 | -12.6794 |
|----|----------|
| 2 | -2.7427 |
| 3 | -1.2357 |
| 4 | -0.7302 |
| 5 | -0.5024 |
| 6 | -0.3829 |
| 7 | -0.3153 |
| 8 | -0.2765 |
| 9 | -0.2563 |
| 10 | 74.1216 |

The first four eigenvectors

| 0.46896D+00 | 0.43190D+00 | 0.40177D+00 | 0.36393D+00 |
|--------------|--------------|--------------|--------------|
| 0.43197D+00 | 0.27443D+00 | 0.76647D-01 | -0.13445D+00 |
| 0.36092D+00 | 0.16900D-01 | -0.31050D+00 | -0.44871D+00 |
| 0.26140D+00 | -0.24679D+00 | -0.44639D+00 | -0.14848D+00 |
| 0.14126D+00 | -0.42050D+00 | -0.22104D+00 | 0.35508D+00 |
| 0.99868D-02 | -0.44089D+00 | 0.18318D+00 | 0.37238D+00 |
| -0.12208D+00 | -0.30054D+00 | 0.43916D+00 | -0.12027D+00 |
| -0.24451D+00 | -0.50605D-01 | 0.33977D+00 | -0.44822D+00 |
| -0.34767D+00 | 0.21778D+00 | -0.34579D-01 | -0.16236D+00 |
| -0.42340D+00 | 0.40676D+00 | -0.38094D+00 | 0.34585D+00 |